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PUBLICATION 575
TECHNICAL BULLETIN 9

ISSUED JULY, 1937
FIRST PRINTING

DOMINION OF CANADA—DEPARTMENT OF AGRICULTURE

MODERN METHODS FOR TESTING A LARGE NUMBER OF VARIETIES

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Published by authority of the Hon. JAMES G. GARDINER, Minister of Agriculture
Ottawa, Canada

MODERN METHODS FOR TESTING A LARGE NUMBER OF VARIETIES[1]

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INTRODUCTION

The field plot designs for testing large numbers of varieties recently developed by Yates (6, 8) should receive the serious consideration of all agronomists concerned with varietal trials. The extent to which they are more efficient than previous methods is of first importance, but the agronomist will also wish to consider their practicability in the field and the additional labour involved in computation. An attempt is made in this paper to discuss these points, and, in addition, fully worked out examples[3] of the methods are included for practice in computation by those wishing to become familiar with the technique.

The principle of error control involved in Pseudo-Factorial and Incomplete Randomized Block experiments may be stated briefly as follows. The number of varieties to be tested is large—we shall say 20 or more—and if these are arranged in ordinary Randomized Blocks, even with long narrow plots, there is certain to be a good deal of uncontrolled variability within the blocks. Another way of stating this is to say that within the blocks a fairly large proportion of the plots are so far apart that there is no correlation between the yields. From studies by various investigators including Harris (2) and more recently by Wiebe (5) with one set of rod row yields, we know that in general there is only a small correlation between pairs of plots that are several plots distant from each other. The ideal block, for the removal of error is such that there is an appreciable correlation between the yields of the outside plots. However when we have 20 or more varieties, due to practical considerations governing the width of the plots, we cannot make up a block containing all of the varieties that meets this requirement. Yates therefore has conceived the idea of making up blocks for error control that contain only a portion of the varieties, and arranging that the distribution of all of the varieties in the various blocks is such that a variety variance can be calculated that is freed from block effects, and an error variance that is appropriate for testing the significance of the variety variance. Blocks made up in this manner may be referred to as *incomplete blocks*. They are usually small in comparison to *complete blocks* that contain all of the varieties, and consequently there is a decided improvement in the efficiency of error control.

Previously there have been various attempts to devise a satisfactory method for testing a large number of varieties all of which for one reason or another have not been completely satisfactory. Student (4) suggested the use of the Semi-Latin Square in which the varieties are arranged in rows and columns as in a Latin Square, the columns being two plots wide so that a square of p^2 dimensions

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[1] Contribution from the Cereal Division, Experimental Farms Branch, Department of Agriculture, Ottawa, Canada. This paper was read before the meeting of the American Society of Agronomy, held in Washington, D.C., November 20, 1936.

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[3] The worked out examples are on a small scale in order to illustrate the methods as briefly as possible. With the exception of Example IV they are not to be considered as typical. All of these examples have been worked on the uniformity data for rod row plots of wheat given by Wiebe (5).

could be used to test $2p$ varieties. Either the rows or columns or both may be made more than one plot wide, in which case the method may be referred to as that of Equalized Random Blocks. Yates (7) has pointed out that designs of this type suffer from a biased error which in the case of Equalized Random Blocks is equal to

$$\frac{n(p-1)}{(np-1)\ (np-2)}\ (E-E'),$$

where n is the number of rows and columns in the square, p is the number of plots in a sub-block, E is the expected variance between sub-blocks and E' is the expected variance within sub-blocks. In general E will be larger than E' and hence the bias will usually be positive. Data have been obtained by the writer (as yet unpublished) indicating that the actual bias in Semi-Latin Squares is frequently significant. However regardless of the existence of a bias it appears to be very unlikely that the Equalized Random Blocks will give results approaching the efficiency of the Pseudo-Factorial methods.

A notable attempt to overcome the difficulties in regard to variability in field trials with a large number of varieties has been made by Richey (3). In general principle this method (adjusting yields to their regression on a moving average) is related to the Pseudo-Factorial method in that it proposes to remove variability within the *complete blocks* as indicated by the yields of the varieties themselves. It would seem rather difficult, however, to provide for this method an exact analytical procedure. This arises in part from the fact that the number of varieties in each moving average group is arbitrarily determined and is not an integral feature of the experimental design.

Various investigators have used the method of systematically placed Controls or Checks in order to remove soil variability within blocks. Yates (6) has given a rather complete discussion of this method and shows that even if correct use is made of the yields of the control plots in adjusting the yields of adjacent plots the results obtained are not likely to be as good as those obtained by the Pseudo-Factorial method. There is another objection to the use of controls which is frequently overlooked. When any one variety is selected for the control plots the assumption is made that the reactions of the varieties tested to changes in soil fertility and other environmental conditions are very similar to the reaction of the control variety. This is not necessarily true. The usual practice is to select for the controls some well-known variety of wide adaptation in the area concerned, and this may lead to serious complications. For example, in Western Canada the most widely grown wheat variety is Marquis, but in a test conducted at Winnipeg in 1935, Marquis yielded 1.9 bushels per acre while a series of varieties resistant to stem rust averaged about 25 bushels per acre. To use Marquis wheat as a control variety in a test of a large group of new rust resistant varieties would obviously be absurd. The only alternative is to select as a control variety one that is almost entirely untried in the area for which the test is being conducted. Analagous cases are likely to arise in any program of breeding new varieties, especially if the new varieties are highly resistant to some condition to which the commonly grown varieties are susceptible. Under these circumstances the experimenter will not wish to take the responsibility of selecting a variety for the controls and will feel much happier if the test can be arranged so that this selection is unnecessary.

A modification of the Randomized Block method sometimes adopted for trials involving a large number of varieties is to arrange the varieties in groups and determine two errors one for comparisons within the groups and one for comparisons between the groups. Supposing that we have 60 varieties divided

into 6 groups and using 4 Randomized Blocks, the analysis is of the following form:

		DF	*Mean Square*
Blocks		3	
Varieties	Between Groups	5	
	Within Groups	54	
Error	Between Groups	15	V_b
	Within Groups	162	V_w
	Total	239	

The variance of the difference between the means of two varieties in the same group, where r is the number of replications, is

$$V \text{ (same group)} = 2V_w/r,$$

while that for comparing two varieties in different groups is

$$V \text{ (different groups)} = \frac{2}{rn}\left[V_b + (n-1)V_w\right],$$

where r is the number of replications and n is the number of varieties in one group. Depending on the size and number of the groups and the shape of the plots, V_b will be found usually to be considerably larger than V_w so that the method resolves itself into sacrificing accuracy in one group of comparisons and gaining it in another group of comparisons.

The difference between the two kinds of variances as illustrated above is usually too great to justify using an average variance for all of the comparisons. It is necessary therefore to have some logical basis for a division of the varieties into groups and frequently this is either very difficult or impossible. It should be noted also that except for the arrangement of the groups in a Latin Square there is no increase in the average precision of the comparisons over the ordinary Randomized Block method.

The next section of this paper is taken up with descriptions of the various Pseudo-Factorial and Incomplete Randomized Block methods and contains a fully worked out example of each type.

TWO DIMENSIONAL PSEUDO-FACTORIAL EXPERIMENTS—TWO EQUAL GROUPS OF SETS

If a set of numbers representing p^2 varieties are arranged in a square as follows:

$$\begin{array}{cccl}
11 & 12 & 13 & \ldots\ldots 1p \\
21 & 22 & 23 & \ldots\ldots 2p \\
31 & 32 & 33 & \ldots\ldots 3p \\
\cdot & \cdot & \cdot & \qquad \cdot \\
\cdot & \cdot & \cdot & \qquad \cdot \\
\cdot & \cdot & \cdot & \qquad \cdot \\
p1 & p2 & p3 & \ldots\ldots pp;
\end{array}$$

the groups in the columns may be taken arbitrarily to represent the factor A as in a factorial experiment, and the groups in the rows may be taken to represent the factor B. The total number of degrees of freedom $(p^2 - 1)$ for the p^2 varieties may therefore be set out as if arising from the main effects and interactions of the two imaginary factors A and B; thus,

Main effects	A	$p - 1$	DF
	B	$p - 1$	DF
Interaction	$A \times B$	$(p - 1)^2$	DF
Total		$p^2 - 1$	$DF.$

Suppose now that the varieties are arranged in *incomplete blocks* each block containing p varieties in a field experiment. In one group making up usually at least two complete replications the varieties are arranged in the blocks according to the rows of the square. Thus the first block will contain the set (11, 12, 13, . . . $1p$) and there will be p such blocks in each replication. In the second group the blocks will be made up according to the columns of the square. Hence the first block of this group will contain the set (11, 21, 31, ... $p1$), and so forth for all of the p columns. The minimum number of replications will be 2 but the actual number of replications of each group which we shall designate by n is limited only by practical considerations. The total number of *incomplete blocks* will be $2np$, and these may be distributed over the field in the manner which the experimenter feels is most convenient for his purpose. All of the blocks for any one group if kept together form a single complete replication and this may be very convenient from the standpoint of making observations on the plots. If the replications representing the first group are on soil quite different in variability from that of the second group, however, there is a possibility of unequal error variance for the two groups, and in order to overcome this it might be necessary to randomize the incomplete blocks of both groups over the whole field or perhaps to keep them together in pairs. However this possibility does not seem to be important in the average test and it should be sufficient to alternate the replications of each group. The only randomization then necessary is of the varieties within the blocks.

On obtaining the yields we are able to arrange them in squares, one square corresponding to each replication and these can be summarized in further squares, one for each group and one for the variety totals. Assuming that we are dealing with an actual case where $p = 4$, and $n = 2$, we can set up a miniature example in algebraic form which is represented diagrammatically in Figure 1. Each variety is represented by a number uv such that u indicates the set to which it belongs in *Group X*, and v the set to which it belongs in *Group Y*.

Group X

11	12	13		11	12	13
21	22	23		21	22	23
31	32	33		31	32	33

Group Y

11	12	13		11	12	13
21	22	23		21	22	23
31	32	33		31	32	33

				$X_{u\cdot}$
	x_{11}	x_{12}	x_{13}	$X_{1\cdot}$
	x_{21}	x_{22}	x_{23}	$X_{2\cdot}$
	x_{31}	x_{32}	x_{33}	$X_{3\cdot}$
$X_{\cdot v}$	$X_{\cdot 1}$	$X_{\cdot 2}$	$X_{\cdot 3}$	$X_{\cdot\cdot}$

				$Y_{u\cdot}$
	y_{11}	y_{12}	y_{13}	$Y_{1\cdot}$
	y_{21}	y_{22}	y_{23}	$Y_{2\cdot}$
	y_{31}	y_{32}	y_{33}	$Y_{3\cdot}$
$Y_{\cdot v}$	$Y_{\cdot 1}$	$Y_{\cdot 2}$	$Y_{\cdot 3}$	$Y_{\cdot\cdot}$

				$T_{u\cdot}$
	T_{11}	T_{12}	T_{13}	$T_{1\cdot}$
	T_{21}	T_{22}	T_{23}	$T_{2\cdot}$
	T_{31}	T_{32}	T_{33}	$T_{3\cdot}$
$T_{\cdot v}$	$T_{\cdot 1}$	$T_{\cdot 2}$	$T_{\cdot 3}$	$T_{\cdot\cdot}$

FIGURE 1. Representation of a Miniature Example of a Two-Dimensional Pseudo-Factorial Experiment with Two Groups of Sets.

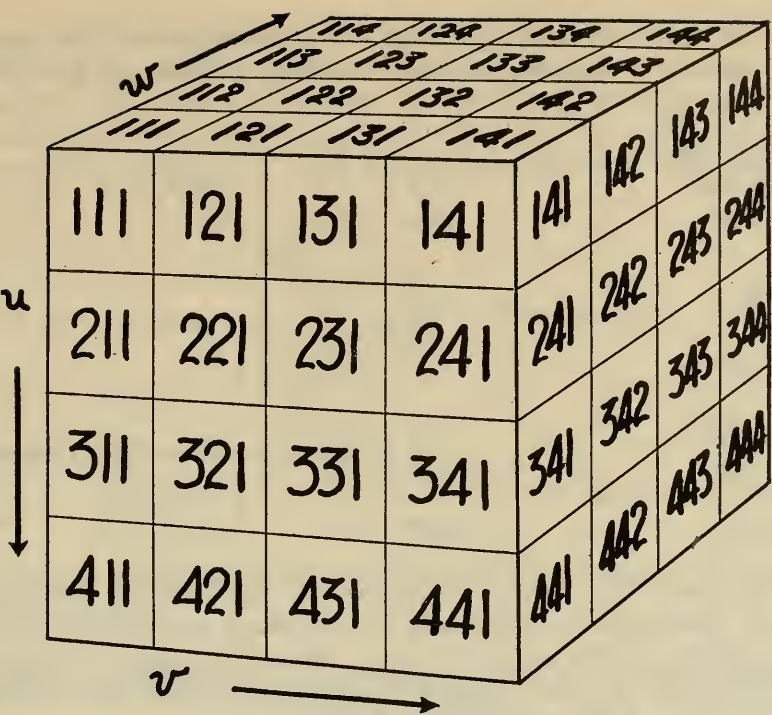

FIGURE 2. A $(4 \times 4 \times 4)$ cube illustrating the principle involved in writing out the sets for a Three Dimensional Pseudo-Factorial Experiment.

Varieties between parallel lines belong to the same set. The first group is *Group X* and the second is *Group Y*. In the variety totals by groups the column and row totals of the squares are represented by the corresponding capital letters X or Y, and the subscripts indicate the variety numbers constant in the given total. Thus $X_{1\cdot}$ means that the varieties totaled have numbers in which the first figure is 1 and a second figure varying from 1 to 3. Now if we examine the row and column totals by groups we note that the totals $X_{1\cdot}$, $X_{2\cdot}$, $X_{3\cdot}$, $Y_{\cdot 1}$, $Y_{\cdot 2}$, and $Y_{\cdot 3}$ contain both variety and block effects. In other words in *Group X* we may assume a factor A which is confounded with block effects, and in *Group Y* a factor B which is confounded with block effects. Hence the A factor must be estimated from *Group Y* and the B factor from *Group X*. Obviously therefore we will have a sum of squares for A represented by

$$\Sigma(Y_{u\cdot}^2)/np - Y_{\cdot\cdot}^2/np^2\,,$$

and for B by

$$\Sigma(X_{\cdot v}^2)/np - X_{\cdot\cdot}^2/np^2\,.$$

The interaction $(A \times \mathrm{B})$ being unconfounded, is estimated from the totals for X and Y combined hence we have the sum of squares for $(A \times B)$ given by

$$\Sigma(T_{uv}^2)/2n - \Sigma(T_{u\cdot}^2)/2np - \Sigma(T_{\cdot v}^2)/2np + T_{\cdot\cdot\cdot}^2/2np^2.$$

The sum of these three sums of squares gives the total for varieties.

Yates gives a direct method of calculating the sum of squares for varieties which is probably quicker than the one used above. Yates' formula in terms of variety and marginal totals is

$$\text{Varieties } (SS) = \Sigma(T_{uv}^2)/2n + \Sigma(X_{u\cdot} - Y_{u\cdot})^2/2np + \Sigma(X_{\cdot v} - Y_{\cdot v})^2/2np - (X_{\cdot\cdot} - Y_{\cdot\cdot})^2/2np^2 - [\,\Sigma(X_{u\cdot}^2) + \Sigma(Y_{\cdot v}^2)]/np.$$

We next calculate the total sum of squares for all of the plots and for the *incomplete blocks*, and obtain the error sum of squares by subtraction. The summarized analysis is of the form

Incomplete Blocks	$2np - 1$
Varieties	$p^2 - 1$
Error	$(p - 1)\ (2np - p - 1)$
Total	$2np^2 - 1$

In order to obtain a clearer picture of the origin of the sums of squares Yates has shown how the degrees of freedom may be set out as follows:

Incomplete Blocks		
Between Groups	1	
Between Sets	$2(p-1)$	$2np-1$
Within Sets	$2(n-1)p$	
Varieties		
A Factor	$p-1$	
B Factor	$p-1$	p^2-1
Interaction	$(p-1)^2$	
Error		
Between Sets	$(p-1)^2$	$(p-1)(2np-p-1)$
Within Sets	$2(n-1)p(p-1)$	
Total	$(2np^2-1)$	

It is of interest to note the origin of the sum of squares for error, and in actual practice it may be desirable to calculate this sum of squares directly in order to have a complete check on the calculations. Referring again to Figure 1 we note that for each square giving the variety totals by groups we have (p^2-1) DF which is apportioned as follows for *Group X*.

Varieties (B factor)	$p-1$
A confounded with blocks	$p-1$
Interaction	$(p-1)^2$,

and similarly for *Group Y*. Now the two interactions represent error to the extent that they are not due to varieties. The latter effect is given by the interaction in the table of totals so that we have by subtraction

$$\text{Interaction } X,\ (p-1)^2 DF + \text{Interaction } Y,\ (p-1)^2 DF$$
$$- \text{Interaction } (X+Y),\ (p-1)^2 DF$$
$$= \text{error between sets } (p-1)^2 DF.$$

The error sum of squares arising from within the sets is due to differences between plots of the same variety within the groups after removal of the differences due to the *incomplete blocks*. Thus for each set there will be $(n-1)(p-1)$ DF giving a total of $2p(n-1)(p-1)$ DF for the $2p$ sets. This portion of the error sum of squares may be calculated directly for the case where $n = 2$ by setting up a table of differences for each group.

In making comparisons between pairs of varieties we cannot use the actual variety totals as they contain block effects. We must make a correction therefore which is based on the yields of the other varieties in the same set. The corrected mean yields t_{uv} are given by

$$t_{uv} = \frac{T_{uv}}{2n} + \frac{1}{2np}(X_{\cdot v} - Y_{\cdot v}) + \frac{1}{2np}(Y_{u\cdot} - X_{u\cdot}) .$$

If a large table of yields is to be corrected it may save time to set up the corresponding portions of the correction in the margins of the table. If we let $C_{\cdot v} = \frac{1}{2np}(X_{\cdot v} - Y_{\cdot v})$ and $C_{u\cdot} = \frac{1}{2np}(Y_{u\cdot} - X_{u\cdot})$, then $C_{\cdot 1}$ will be the portion of the correction to be applied to all of the variety means in the first column, and $C_{1\cdot}$ will be the portion applied to all of the means in the first row.

In this as well as in all of the other Pseudo-Factorial arrangements the error variance must be multiplied by a factor depending on the type of experiment to give the variance for comparing two varieties by their corrected means.

The error variance s^2 furnishes directly a test of the significance of variety differences as a whole but in order to compare any two varieties we must use the corrected means and the sum of squares of these values is not the true sum of squares of the varieties. In comparing varieties having a set in common, if s^2 is the error variance, the variance of the difference between the corrected variety means will be

$$V(t_{21} - t_{11}) = \frac{s^2}{n}\left(\frac{p+1}{p}\right).$$

For two varieties not having a set in common the variance of the difference is

$$V(t_{22} - t_{11}) = \frac{s^2}{n}\left(\frac{p+2}{p}\right).$$

The mean variance of all comparisons is

$$V_m = \frac{s^2}{n}\left(\frac{p+3}{p+1}\right),$$

and when p is not too small we may use the latter variance for all comparisons without appreciable error.

Example I.—Two Dimensional Pseudo-Factorial Experiment with Two Groups of Sets

Varieties in each set (p) = 5.

Varieties (v) = p^2 = 25 designated by numbers (uv) as follows:

11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45
51	52	53	54	55

Sets (s) = $2p$ = 10, written out by taking 5 sets according to the rows of the above square for *Group X*, and 5 sets according to the columns for *Group Y*.

Replications of each group	(n)	= 2.
Complete replications	(r)	= $2n$ = 4.
Total number of blocks	(b)	= $2np$ = 20.
Total number of plots	(N)	= $2np^2$ = 100.

Table 1 gives the position of the varieties in the field after randomization of the varieties within the blocks, and the corresponding plot yields and block totals. Note that the sets have been kept together to form complete replications, and that the varieties have been randomized within the blocks. The blocks are also arranged at random within the replications but this was unnecessary, and it would have been more correct to have alternated the X and Y groups.

Table 2 contains the variety yields collected first by groups and then for both groups. All marginal totals must be obtained and designated according to group and set. Thus the totals for the sets of *Group X* are designated by $X_{u\cdot}$ and the totals across the sets by $X_{\cdot v}$. At the foot of the table are the differences between the corresponding marginal means of X and Y to be used in calculating the variety sum of squares by one method and in calculating the corrected variety means.

By the shortest method the variety sum of squares is calculated as follows:

$\Sigma(T_{uv}^2)/2n$	=	1,961,637.50	
$\Sigma(X_{u\cdot} - Y_{u\cdot})^2/2np$	=	81,162.50	
$\Sigma(X_{\cdot v} - Y_{\cdot v})^2/2np$	=	117,817.50	
$-(X_{\cdot\cdot} - Y_{\cdot\cdot})^2/2np^2$	=	−51,076.00	
$-[\Sigma(X_{u\cdot}^2) + \Sigma(Y_{\cdot v}^2)]/np$	=	−2,058,800.00	(Groups + Sets + Mean)
Varieties = Sum	=	50,741.50	

By the other method we obtain

A factor	$\Sigma(Y_{u\cdot}^2)/np - Y_{\cdot\cdot}^2/np^2$	=	9,658.0
B factor	$\Sigma(X_{\cdot v}^2)/np - X_{\cdot\cdot}^2/np^2$	=	6,422.0
Interaction ($A \times B$)	$\Sigma(T_{uv}^2)/2n - \Sigma(T_{u\cdot}^2)/2np - \Sigma(T_{\cdot v}^2)/2np + T_{\cdot\cdot}^2/2np^2$	=	34,661.5
	Varieties = Sum	=	50,741.50

The total sum of squares for all plots is 630,266.00 and for blocks is 467,586.00. Having obtained these we can set up the analysis of variance.

Analysis of Variance

Two dimensional—two groups of sets

	SS	*DF*	*MS*	*F*	5% *pt.*
Blocks	467,586.00	19	24,609.8	12.3	1.78
Varieties	50,741.50	24	2,114.2	1.15	1.72
Error	111,938.50	56	1,998.9		
Total	630,266.00	99			

In order to obtain the corrected variety yields we calculate

$$C_{\cdot v} = \frac{1}{2np}(X_{\cdot v} - Y_{\cdot v}) \text{ for } v = 1, 2, 3, 4, 5$$

and

$$C_{u\cdot} = \frac{1}{2np}(Y_{u\cdot} - X_{u\cdot}) \text{ for } u = 1, 2, 3, 4, 5.$$

These are entered in the margins of a (5×5) table as illustrated in Table 3, and added to the actual means of corresponding cells in the table.

To make comparisons between the corrected means we may take into consideration whether or not the varieties being compared occur in the same set. To compare varieties 21 and 22 for example we calculate the variance according to the formula

$$V(t_{21} - t_{22}) = \frac{s^2}{n}\left(\frac{p+1}{p}\right) = \left(\frac{1998.9}{2} \times \frac{6}{5}\right) = 1199.3$$

$$SE(t_{21} - t_{22}) = \sqrt{1199.3} = 34.63$$

$$t^* = \frac{161.50 - 123.75}{34.63} = 1.09\,.$$

To compare varieties 11 and 54 we would have

$$V(t_{11} - t_{54}) = \frac{s^2}{n}\left(\frac{p+2}{p}\right) = \left(\frac{1998.9}{2} \times \frac{7}{5}\right) = 1399.23$$

$$SE(t_{11} - t_{54}) = \sqrt{1399.23} = 37.41$$

$$t = \frac{135.25 - 170.25}{37.41} = .94.$$

We would obviously not be very far wrong even with a p value as low as five to use for all comparisons the mean variance for the difference between two varieties. This would be

$$V_m = \frac{s^2}{n}\left(\frac{p+3}{p+1}\right) = \left(\frac{1998.9}{2} \times \frac{8}{6}\right) = 1332.6$$

$$SE_m = \sqrt{1332.6} = 36.50.$$

*The t used here is of course the statistic defined by R. A. Fisher in *Statistical Methods for Research Workers*.

TABLE 1.—POSITION OF VARIETIES IN THE FIELD AND CORRESPONDING PLOT YIELDS. TWO DIMENSIONAL PSEUDO-FACTORIAL EXPERIMENT WITH TWO GROUPS OF SETS

Set No.	Var. No.	Yield	Var. No.	Yield	Var. No.	Yield	Var. No.	Yield	Var. No.	Yield	Block Totals
1y	31	215	21	300	51	255	41	185	11	145	1100
2y	22	150	12	50	52	45	32	105	42	155	505
5y	55	125	35	30	15	65	25	130	45	55	405
4y	14	85	34	55	54	110	24	130	44	40	420
3y	53	45	43	45	13	60	23	15	33	−5	160
1y	11	210	21	290	41	325	31	230	51	220	1275
2y	12	310	32	230	22	155	52	195	42	245	1135
5y	15	315	45	215	55	160	25	285	35	230	1205
3y	53	185	43	220	33	175	13	275	23	185	1040
4y	14	130	24	190	34	160	44	110	54	155	745
1x	14	140	15	165	11	265	13	150	12	180	900
4x	41	190	42	135	45	100	43	145	44	205	775
3x	33	250	31	150	35	150	34	195	32	155	900
2x	22	75	21	105	25	130	23	180	24	90	580
5x	55	40	54	155	53	65	52	60	51	40	360
5x	55	115	54	185	53	240	51	120	52	125	785
1x	11	145	13	105	14	50	15	130	12	135	565
3x	32	150	33	115	34	60	35	110	31	25	460
2x	21	5	24	65	25	70	23	60	22	20	220
4x	41	30	42	50	43	35	45	20	44	50	185
										Grand Total =	13,720

TABLE 2.—YIELDS OF VARIETIES BY GROUPS, AND TOTAL YIELDS FOR BOTH GROUPS

Values of x_{uv}

		v = 1	2	3	4	5	$X_{u\cdot}$
Group X	u = 1	410	315	255	190	295	1465
	2	110	95	240	155	200	800
	3	175	305	365	255	260	1360
	4	220	185	180	255	120	960
	5	160	185	305	340	155	1145
	$X_{\cdot v}$	1075	1085	1345	1195	1030	5730 = $X_{\cdot\cdot}$

Values of y_{uv}

		v = 1	2	3	4	5	$Y_{u\cdot}$
Group Y	u = 1	355	360	335	215	380	1645
	2	590	305	200	320	415	1830
	3	445	335	170	215	260	1425
	4	510	400	265	150	270	1595
	5	475	240	230	265	285	1495
	$Y_{\cdot v}$	2375	1640	1200	1165	1610	7990 = $Y_{\cdot\cdot}$

Values of T_{uv}

		v = 1	2	3	4	5	$T_{u\cdot}$
Group X + Group Y	u = 1	765	675	590	405	675	3110
	2	700	400	440	475	615	2630
	3	620	640	535	470	520	2785
	4	730	585	445	405	390	2555
	5	635	425	535	605	440	2640
	$T_{\cdot v}$	3450	2725	2545	2360	2640	13720 = $T_{\cdot\cdot}$

v	$X_{\cdot v} - Y_{\cdot v}$	u	$Y_{u\cdot} - X_{u\cdot}$
1	−1300	1	180
2	− 555	2	1030
3	145	3	65
4	30	4	635
5	− 580	5	350
$(X_{\cdot\cdot} - Y_{\cdot\cdot})$ =	−2260	$(Y_{\cdot\cdot} - X_{\cdot\cdot})$ =	2260

TABLE 3.—CALCULATION OF CORRECTED VARIETY MEANS (t_{uv})

u \ v	1	2	3	4	5	$C_{u\cdot}$
1	135.25	150.00	163.75	111.75	148.75	9.00
2	161.50	123.75	168.75	171.75	176.25	51.50
3	93.25	135.50	144.25	122.25	104.25	3.25
4	149.25	150.25	150.25	134.50	100.25	31.75
5	111.25	96.00	158.50	170.25	98.50	17.50
$C_{\cdot v}$	−65.00	−27.75	7.25	1.50	−29.00	0

$$C_{\cdot 1} = -1300/20 = -65.00$$

$$C_{1\cdot} = 180/20 = 9.00$$

Two Dimensional Pseudo-Factorial Experiment—Three Groups of Sets

A possible criticism of the Pseudo-Factorial method with two groups of sets is that there is too great a discrepancy between the estimates of the error variance for comparing varieties in the same and in different sets. This can be overcome by increasing the number of groups and we shall see later that by increasing the number of groups to the limit we arrive at a point where the variance for all comparisons is the same. The type with three groups of sets is therefore transitional between that with two groups and the limiting type to be discussed later.

In order to set up the three groups of sets such that any one variety does not occur twice with any other variety it is sufficient to write down the numbers for the varieties in a square starting in the same manner as for two groups of sets using the first figure to represent the rows and the second figure the columns. We then write the third set of figures in the diagonals. For $p = 4$ we get the square given below.

111	124	133	142
212	221	234	243
313	322	331	344
414	423	432	441

We can now proceed to write out the sets:

Group X				*Group Y*				*Group Z*			
111	124	133	142	111	212	313	414	111	221	331	441
212	221	234	243	124	221	322	423	212	322	432	142
313	322	331	344	133	234	331	432	313	423	133	243
414	423	432	441	142	243	344	441	414	124	234	344

This gives us 12 sets in all or in general $3p$ and if each group is replicated n times we have a total of $3np$ *incomplete blocks.*

The next step is to distribute the *incomplete blocks* over the field, and, if it is more convenient, keeping the groups together to form complete replications. For the example given below the plot yields and the corresponding numbers as they occur when arranged at random over the field are given in Table 4. Again the blocks could have been arranged systematically instead of at random within each group.

Proceeding to the calculation of sums of squares the first step is to set up a table similar to Table 5. This gives the totals by groups and the complete variety totals. The latter are set up in two ways so as to give the three sets of marginal totals $T_{u\cdot\cdot}$, $T_{\cdot v\cdot}$, and $T_{\cdot\cdot w}$. The totals represented by $X_{u\cdot\cdot}$, $Y_{\cdot v\cdot}$, and

$Z_{..w}$ are obviously the totals for sets. The sum of squares for (groups + sets + varieties + mean) is given by:

$$\frac{\Sigma(T_{uvw}^2)}{3n} + \frac{\Sigma(3X_{u..} - T_{u..})^2 + \Sigma(3Y_{.v.} - T_{.v.})^2 + \Sigma(3Z_{..w} - T_{..w})^2}{6np}$$
$$- \frac{(3X_{...} - T_{...})^2 + (3Y_{...} - T_{...})^2 + (3Z_{...} - T_{...})^2}{18np^2}.$$

Since the values in the second term represented by $(3X_{u..} - T_{u..})$, etc., will be used again in determining the corrected variety means, it is just as well to tabulate them. Note also that $\Sigma(3X_{u..} - T_{u..}) = (3X_{...} - T_{...})$, etc., so that after the tabulation of the values for the second term, totalling for each group gives the values for the third term. Then the sum of squares for (groups + sets + mean) is determined from

$$\frac{\Sigma(X_{u..}^2) + \Sigma(Y_{.v.}^2) + \Sigma(Z_{..w}^2)}{np}.$$

Subtracting this from that for (groups + sets + varieties + mean) we obtain the sum of squares for varieties. Finally we require only the sum of squares for blocks and for the total of all plots, in order to obtain the sum of squares for error by subtraction. The sum of squares for blocks will of course be calculated from the block totals as given in Table 4.

The partition of the degrees of freedom for the analysis of variance will be:

Blocks	$3np - 1$
Varieties	$p^2 - 1$
Error	$(p - 1)\ (3np - p - 1).$
Total	$3np^2 - 1$

The DF may of course be broken down as follows into the various components as for two groups of sets, but this is unnecessary in routine analysis.

Blocks	Between Groups	2
	Between Sets	$3(p - 1)$
	Within Sets	$3p(n - 1)$
Varieties		$(p^2 - 1)$
Error	Between Groups	$(p - 1)\ (2p - 1)$
	Within Groups	$3p(p - 1)\ (n - 1)$
	Total	$(3np^2 - 1)$

In order to compare pairs of varieties we must calculate the corrected variety means. These are represented by t_{uvw} and are given by:

$$t_{uvw} = \frac{T_{uvw}}{3n} - \frac{(3X_{u..} - T_{u..}) + (3Y_{.v.} - T_{.v.}) + (3Z_{..w} - T_{..w})}{6np}$$

If s^2 is the error variance, the variance of the difference between the means of varieties occurring in the same set of one of the groups is

$$V(t_{uvw} - t_{uv'w'}) = \frac{2s^2}{3n}\left(1 + \frac{1}{p}\right)$$

and for varieties not occurring in the same set

$$V(t_{uvw} - t_{u'v'w'}) = \frac{2s^2}{3n}\left(1 + \frac{3}{2p}\right)$$

The average variance of all comparisons is

$$V_m = \frac{2s^2}{3n}\left(\frac{p + 2\frac{1}{2}}{p + 1}\right)$$

Since in nearly all cases the corrected variety means must be worked out, an alternative method for calculating the variety sum of squares is suggested using the corrected variety means. If the corrected variety means are averaged in three ways: (1) all those containing the same value of u giving quantities represented by $t_{u\cdot\cdot}$; (2) all those containing the same value of v, giving $t_{\cdot v\cdot}$; and (3) all those containing the same value of w, giving $t_{\cdot\cdot w}$; then the variety sum of squares is given by

$$\Sigma(t_{uvw} \cdot T_{uvw}) - [\Sigma(X_{u\cdot\cdot} \cdot t_{u\cdot\cdot}) + \Sigma(Y_{\cdot v\cdot} \cdot t_{\cdot v\cdot}) + \Sigma(Z_{\cdot\cdot w} \cdot t_{\cdot\cdot w})] .$$

This furnishes a very useful check on the previous method of calculating the variety sum of squares, but is not an exact check unless the corrected means are carried out to a sufficient number of decimal places.

Example II.—Two Dimensional Pseudo-Factorial Experiment with Three Groups of Sets

Varieties in each set (p) = 4.

Varieties (v) = p^2 = 16, designated by numbers uvw as follows:

111	124	133	142
212	221	234	243
313	322	331	344
414	423	432	441

where u represents the set of *Group X*, v the set of *Group Y*, and w the set of *Group Z*. This square is made up by writing the first number to represent the rows, the second number the columns, and the third number is written in on the diagonals.

Sets (s) = $3p$ = 12; two sets written out from the rows and columns of the square and the third set by taking the numbers in the diagonals.

Replications of each group	(n)	= 2.
Complete replications	(r)	= $3n$ = 6.
Total number of blocks	(b)	= $3np$ = 24.
Total number of plots	(N)	= $3np^2$ = 96.

Table 4 gives the position of the varieties in the field after the randomization of the varieties within the blocks, and the corresponding plot yields and block totals. The sets have been kept together to form complete replications, and it was not essential to arrange them at random within each group.

In Table 5 the yields of the varieties have been collected by groups and for all three groups. The complete variety totals have been arranged in two ways so that the marginal totals $T_{u\cdot\cdot}$ and $T_{\cdot v\cdot}$ are given by one arrangement and $T_{\cdot\cdot w}$ and $T_{\cdot v\cdot}$ by the second arrangement.

The variety sum of squares can be calculated directly from Table 5 according to the following scheme:

$$\Sigma(T_{uvw}^2)/3n \qquad = \qquad 4,263,412.50$$

$$\left.\begin{array}{l} \Sigma(3X_{u\cdot\cdot} - T_{u\cdot\cdot})^2/6np \\ \Sigma(3Y_{\cdot v\cdot} - T_{\cdot v\cdot})^2/6np \\ \Sigma(3Z_{\cdot\cdot w} - T_{\cdot\cdot w})^2/6np \end{array}\right\} \qquad = \qquad 445,984.37$$

$$\left.\begin{array}{l} -(3X_{\cdot\cdot\cdot} - T_{\cdot\cdot\cdot})^2/18np^2 \\ -(3Y_{\cdot\cdot\cdot} - T_{\cdot\cdot\cdot})^2/18np^2 \\ -(3Z_{\cdot\cdot\cdot} - T_{\cdot\cdot\cdot})^2/18np^2 \end{array}\right\} \qquad = - \qquad 130,903.12$$

$$-[\Sigma(X_{u\cdot\cdot}^2) + \Sigma(Y_{\cdot v\cdot}^2) + \Sigma(Z_{\cdot\cdot w}^2)]/np = -4,487,984.38 \text{ (Groups + Sets + Mean)}$$

$$\text{Varieties } (SS) = \text{Sum} = \qquad 90,509.37$$

Again an alternative method of getting the sum of squares for varieties is suggested if it is certain that the corrected variety means are to be calculated.

Having gotten these, they can be used to calculate the sum of squares for the varieties. First the corrected means are calculated from the formula

$$t_{uvw} = \frac{T_{uvw}}{3n} + C_{u\cdot\cdot} + C_{\cdot v\cdot} + C_{\cdot\cdot w},$$

where

$$C_{u\cdot\cdot} = (T_{u\cdot\cdot} - 3X_{u\cdot\cdot})/6np$$
$$C_{\cdot v\cdot} = (T_{\cdot v\cdot} - 3Y_{\cdot v\cdot})/6np$$
$$C_{\cdot\cdot w} = (T_{\cdot\cdot w} - 3Z_{\cdot\cdot w})/6np.$$

Table 6 gives the actual means $T_{uvw}/3n$, in the first section, and in the margins the values of the correction terms. Note that in applying the correction terms, $C_{u\cdot\cdot}$ and $C_{\cdot v\cdot}$ are in the corresponding row and column of the table but that $C_{\cdot\cdot w}$ must be picked out from the value of w for the variety. Thus

$$t_{124} = 173.333 + 31.250 - 44.375 + 12.187 = 172.395.$$

Having prepared the table of corrected means they are averaged as in the next section of Table 6 to give $t_{u\cdot\cdot}$, $t_{\cdot v\cdot}$ and $t_{\cdot\cdot w}$. To get $t_{\cdot\cdot w}$ we average the corrected means according to the value of w, or in other words along the diagonals of the square.

The variety sum of squares is then given by

$$\text{Varieties } (SS) = \Sigma(t_{uvw} \cdot T_{uvw}) - \Sigma(t_{u\cdot\cdot} \cdot X_{u\cdot\cdot}) - \Sigma(t_{\cdot v\cdot} \cdot Y_{\cdot v\cdot}) - \Sigma(t_{\cdot\cdot w} \cdot Z_{\cdot\cdot w})$$

For the present example this gives

$$4{,}259{,}920.66 - 4{,}169{,}411.19 = 90{,}509.47,$$

a close check on the first method.

After calculating the total and block sums of squares we have the following analysis of variance:

Analysis of Variance

Two dimensional experiment—three groups of sets

	SS	*DF*	*MS*	*F*	5% *pt.*
Blocks	539,585.16	23	23,460.2		
Varieties	90,509.37	15	6,034.0	1.55	1.86
Error	221,646.88	57	3,888.5		
Total	851,741.41	95			

In making direct comparisons the varieties may be classified according to whether they differ in two sets or three sets. Thus for the varieties 111 and 124 differing in two sets the variance of a difference between the corrected means is

$$V(t_{111} - t_{124}) = \frac{2s^2}{3n}\left(1 + \frac{1}{p}\right) = \left(\frac{2 \times 3888.5}{6}\right)\left(\frac{5}{4}\right) = 1620.2$$

$$SE(t_{111} - t_{124}) = \sqrt{1620.2} = 40.25$$

$$t = \frac{266.4 - 172.4}{40.25} = 2.34$$

For varieties 111 and 322 differing in three sets

$$V(t_{111} - t_{322}) = \frac{2s^2}{3n}\left(1 + \frac{3}{2p}\right) = \left(\frac{2 \times 3888.5}{6}\right)\left(\frac{11}{8}\right) = 1782.2$$

$$SE(t_{111} - t_{322}) = \sqrt{1782.2} = 42.22$$

$$t = \frac{266.4 - 213.2}{42.22} = 1.26$$

Table 4.—Position of the varieties in the field after randomization and corresponding plot yields. Two dimensional pseudo-factorial experiment with three groups of sets

					Block Totals					Block Totals
Var. No's.	(124)	(423)	(322)	(221)		(221)	(423)	(322)	(124)	
Yields	315	370	360	265	1310	195	310	315	215	1035
	(142)	(344)	(441)	(243)		(331)	(432)	(234)	(133)	
	355	345	245	185	1130	330	270	290	95	985
Group Y										
	(414)	(313)	(111)	(212)		(414)	(111)	(212)	(313)	
	160	285	355	240	1040	140	330	410	235	1115
	(331)	(133)	(432)	(234)		(243)	(142)	(344)	(441)	
	325	315	300	240	1180	255	375	305	255	1190
					4660					4325
	(234)	(221)	(212)	(243)		(441)	(414)	(423)	(432)	
	180	255	290	285	1010	180	275	290	155	900
	(344)	(331)	(313)	(322)		(331)	(344)	(313)	(322)	
	270	185	150	55	660	180	160	120	70	530
Group X										
	(124)	(111)	(142)	(133)		(142)	(111)	(124)	(133)	
	50	210	265	185	710	100	100	170	65	435
	(423)	(441)	(414)	(432)		(212)	(221)	(243)	(234)	
	130	215	155	95	595	55	145	40	35	275
					2975					2140
	(441)	(111)	(331)	(221)		(221)	(111)	(441)	(331)	
	215	300	255	185	955	210	290	325	230	1055
	(414)	(344)	(234)	(124)		(212)	(322)	(432)	(142)	
	145	150	50	45	390	220	310	230	155	915
Group Z										
	(423)	(133)	(313)	(243)		(234)	(124)	(414)	(344)	
	105	155	125	30	415	195	245	315	215	970
	(322)	(212)	(432)	(142)		(133)	(313)	(423)	(243)	
	65	130	55	85	335	160	285	230	185	860
					2095					3800

TABLE 5.—VARIETY TOTALS BY GROUPS AND FOR GROUPS COMBINED

	Values of x_{uvw}				$X_{u..}$		Values of T_{uvw}				$T_{u..}$
Var. No's.	(111)	(124)	(133)	(142)			(111)	(124)	(133)	(142)	
Yields	310	220	250	365	1145		1585	1040	975	1335	4935
	(212)	(221)	(234)	(243)			(212)	(221)	(234)	(243)	
	345	400	215	325	1285		1345	1255	990	980	4570
Group X	(313)	(322)	(331)	(344)			(313)	(322)	(331)	(344)	
	270	125	365	430	1190		1200	1175	1505	1445	5325
	(414)	(423)	(432)	(441)			(414)	(423)	(432)	(441)	
	430	420	250	395	1495		1190	1435	1105	1435	5165
$X_{.v.}$	1400	1145	990	1580	5115 $= X_{...}$	$T_{.v.}$	5320	4905	4575	5195	19995 $= T_{...}$

	Values of y_{uvw}				$Y_{.v.}$		Values of T_{uvw}				$T_{..w}$
	(111)	(212)	(313)	(414)			(111)	(221)	(331)	(441)	
	685	650	520	300	2155		1585	1255	1505	1435	5780
	(124)	(221)	(322)	(423)			(212)	(322)	(432)	(142)	
	530	460	675	680	2345		1345	1175	1105	1335	4960
Group Y	(133)	(234)	(331)	(432)			(313)	(423)	(133)	(243)	
	410	530	655	570	2165		1200	1435	975	980	4590
	(142)	(243)	(344)	(441)			(414)	(124)	(234)	(344)	
	730	440	650	500	2320		1190	1040	990	1445	4665
$Y_{u..}$	2355	2080	2500	2050	8985 $= Y_{...}$	$T_{.v.}$	5320	4905	4575	5195	19995 $= T_{...}$

	Values of z_{uvw}				$Z_{..w}$
	(111)	(221)	(331)	(441)	
	590	395	485	540	2010
	(212)	(322)	(432)	(142)	
	350	375	285	240	1250
Group Z	(313)	(423)	(133)	(243)	
	410	335	315	215	1275
	(414)	(124)	(234)	(344)	
	460	290	245	365	1360
$Z_{.v.}$	1810	1395	1330	1360	5895 $= Z_{...}$

Table 6.—Calculation of corrected variety means and alternative method for variety sum of squares

					$C_{u..}$
Var. No's.	(511)	(124)	(133)	(142)	
Yields	264.167	173.333	162.500	222.500	+ 31.250
	(212)	(221)	(234)	(243)	
	224.167	209.167	165.000	163.333	+ 14.896
	(313)	(322)	(331)	(344)	
	200.000	195.833	250.833	240.833	+ 36.562
	(414)	(423)	(432)	(441)	
	198.333	239.167	184.167	239.167	+ 14.167
$C_{.v.}$	−23.854	−44.375	−40.000	−36.771	
$C_{..w}$	− 5.208	+25.208	+15.938	+12.187	

u	$3X_{u..}-T_{u..}$	v	$3Y_{.v.}-T_{.v.}$	w	$3Z_{..w}-T_{..w}$
1	−1500	1	1145	1	250
2	− 715	2	2130	2	−1210
3	−1755	3	1920	3	− 765
4	− 680	4	1765	4	− 585

$(3X_{...}-T_{...}) = -4650$ $\quad (3Y_{...}-T_{...}) = 6960$ $\quad (3Z_{...}-T_{...}) = -2310$

					$t_{u..}$	w	$t_{..w}$
	(111)	(124)	(133)	(142)			
	266.355	172.395	169.688	242.187	212.656	1	223.594
	(212)	(221)	(234)	(243)			
	240.417	174.480	152.083	157.396	181.094	2	219.844
	(313)	(322)	(331)	(344)			
	228.646	213.228	242.187	252.811	234.218	3	195.157
	(414)	(423)	(432)	(441)			
	200.833	224.897	183.542	211.355	205.157	4	194.530
$t_{.v.}$	234.063	196.250	186.875	215.937			

u	$t_{u..}$	$X_{u..}$	v	$t_{.v.}$	$Y_{.v.}$	w	$t_{..w}$	$Z_{..w}$
1	212.656	1145	1	234.063	2155	1	223.594	2010
2	181.094	1285	2	196.250	2345	2	219.844	1250
3	234.218	1190	3	186.875	2165	3	195.157	1275
4	205.157	1495	4	215.937	2320	4	194.530	1360

$$\Sigma(t_{uvw} \cdot T_{uvw}) = 4{,}259{,}920.66$$

$$\left.\begin{array}{l} -\,\Sigma(t_{u..} \cdot X_{u..}) \\ -\,\Sigma(t_{.v.} \cdot Y_{.v.}) \\ -\,\Sigma(t_{..w} \cdot Z_{..w}) \end{array}\right\} = -4{,}169{,}411.19$$

$$\text{Varieties } (SS) = 90{,}509.47$$

THREE DIMENSIONAL PSEUDO-FACTORIAL EXPERIMENT—THREE GROUPS OF SETS

Using the methods previously described the number of varieties was necessarily a perfect square. Now if we have an unusually large number of varieties, say 150 or more, the blocks must contain 12 or more plots and again become somewhat large for the maximum control of error. An alternative method suggested by Yates for such trials is based on the arrangement of the varieties in sets made up from a cube in which the numbers *uvw* designating the varieties represent a given position in the cube. A $(4 \times 4 \times 4)$ cube of this type is illustrated in Figure 2. From this cube we can write down the three groups of sets at once. The first group results from slicing the cube in one direction, the second from slicing in another direction, and the third from slicing in the third direction. Each complete slice gives 4 sets in the $(4 \times 4 \times 4)$ cube or p sets in a $(p \times p \times p)$ cube, so that altogether we have $3p^2$ sets.

The actual sets are written out in Table 7 for the $(4 \times 4 \times 4)$ cube. Any one variety is denoted by the three numbers uvw, and note that vw are constant in the sets of *Group X*, uw in *Group Y*, and uv in *Group Z*. Merely by expanding or contracting the $(4 \times 4 \times 4)$ arrangement given here the sets may be written out for any other arrangement. For example in writing down the sets for a $(5 \times 5 \times 5)$ arrangement we would start as shown at the foot of Table 7.

TABLE 7.—SETS FOR A $(4\times4\times4)$ PSEUDO-FACTORIAL EXPERIMENT

Group X(.vw)				*Group Y(u.w)*				*Group Z(uv.)*			
111	211	311	411	111	121	131	141	111	112	113	114
112	212	312	412	211	221	231	241	121	122	123	124
113	213	313	413	311	321	331	341	131	132	133	134
114	214	314	414	411	421	431	441	141	142	143	144
121	221	321	421	112	122	132	142	211	212	213	214
122	222	322	422	212	222	232	242	221	222	223	224
123	223	323	423	312	322	332	342	231	232	233	234
124	224	324	424	412	422	432	442	241	242	243	244
131	231	331	431	113	123	133	143	311	312	313	314
132	232	332	432	213	223	233	243	321	322	323	324
133	233	333	433	313	323	333	343	331	332	333	334
134	234	334	434	413	423	433	443	341	342	343	344
141	241	341	441	114	124	134	144	411	412	413	414
142	242	342	442	214	224	234	244	421	422	423	424
143	243	343	443	314	324	334	344	431	432	433	434
144	244	344	444	414	424	434	444	441	442	443	444

Group X(.vw)					*Group Y(u.w)*					*Group Z(uv.)*				
111	211	311	411	511	111	121	131	141	151	111	112	113	114	115
112	212	312	412	512	211	221	231	241	251	121	122	123	124	125
113	213	313	413	513	311	321	331	341	351	131	132	133	134	135
–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

etc.

After writing down the sets we decide on (n) the number of replications of each group and proceed to distribute the blocks over the field according to any convenient system.

The calculations are best carried out in tabular form as in Table 9. The data are first collected by groups so that in the table the yield of any one variety in one group will be a total of n plots. The various marginal totals are obtained as indicated in three directions and it will be noted that $X_{\cdot vw}$, $Y_{u\cdot w}$ and $Z_{uv\cdot}$ represent the totals for the sets. The complete variety totals represented by T_{uvw} are entered next and all of the marginal totals of these obtained. This brings us to the calculation of the corrected variety means to be used in comparing varieties and calculating the variety sum of squares. The most convenient formula for the corrected means is

$$t_{uvw} = \frac{T_{uvw}}{3n} + C_{\cdot vw} + C_{u\cdot w} + C_{uv\cdot} \,,$$

where

$$C_{\cdot vw} = \frac{1}{6np^2}\left(pT_{\cdot vw} - 3pX_{\cdot vw} - T_{\cdot v\cdot} + 3Y_{\cdot v\cdot}\right)$$

$$C_{u\cdot w} = \frac{1}{6np^2}\left(pT_{u\cdot w} - 3pY_{u\cdot w} - T_{\cdot\cdot w} + 3Z_{\cdot\cdot w}\right)$$

$$C_{uv\cdot} = \frac{1}{6np^2}\left(pT_{uv\cdot} - 3pZ_{uv\cdot} - T_{u\cdot\cdot} + 3X_{u\cdot\cdot}\right) .$$

The correction terms $C_{\cdot vw}$, $C_{u\cdot w}$ and $C_{uv\cdot}$ can be calculated first and entered in the margins. At this point the calculations can be checked by adding all of the C's. These should total to zero within the limits of the errors introduced by dropping decimal figures. The corrected means are then obtained by adding to the actual means $(T_{uvw}/3n)$ the corresponding three correction terms. Finally the corrected means are averaged in three directions to give the constants $t_{\cdot vw}$, $t_{u\cdot w}$ and $t_{uv\cdot}$.

The sum of squares for varieties is now given by

$$\Sigma(t_{uvw}\cdot T_{uvw}) - [\,\Sigma(X_{\cdot vw}\cdot t_{\cdot vw}) + \Sigma(Y_{u\cdot w}\cdot t_{u\cdot w}) + \Sigma(Z_{uv\cdot}\cdot t_{uv\cdot})]$$

The sum of squares for blocks can be obtained directly from the block totals and the error sum of squares by subtraction from the total sum of squares.

Pairs of varieties can be classified in three ways for comparison by means of the variance of the mean difference. These classes are as follows as indicated by the subscript numbers.

$$(1)\quad V(t_{211} - t_{111}) = \frac{2s^2}{3np^2}(p^2 + p + 1)$$

$$(2)\quad V(t_{122} - t_{111}) = \frac{s^2}{3np^2}(2p^2 + 3p + 4)$$

$$(3)\quad V(t_{222} - t_{111}) = \frac{s^2}{3np^2}(2p^2 + 3p + 6)\,.$$

Finally the mean variance of all comparisons is

$$V_m = \frac{s^2}{3n}\left(\frac{2p^2 + 5p + 11}{p^2 + p + 1}\right).$$

TABLE 8.—POSITION OF VARIETIES IN THE FIELD AND CORRESPONDING PLOT YIELDS. THREE DIMENSIONAL PSEUDO-FACTORIAL EXPERIMENT WITH THREE GROUPS OF SETS

Set No.	Var.	Yield	Var.	Yield	Var.	Yield	Block Totals	Set No.	Var.	Yield	Var.	Yield	Var.	Yield	Block Totals
2*x*	212	315	312	370	112	360	1045	4*y*	122	195	112	310	132	315	820
5*y*	222	265	232	355	212	345	965	6*z*	233	215	231	330	232	270	815
6*y*	322	245	312	185	332	160	590	9*y*	333	290	313	95	323	140	525
8*y*	223	285	233	355	213	240	880	7*x*	231	330	131	410	331	235	975
2*y*	211	325	221	315	231	300	940	6*y*	312	255	322	375	332	305	935
5*x*	122	240	322	220	222	350	810	4*x*	121	255	321	235	221	230	720
2*x*	212	360	312	230	112	225	815	5*y*	232	275	222	245	212	140	660
3*y*	331	270	311	255	321	170	695	5*z*	223	270	222	230	221	135	635
6*x*	323	175	123	290	223	330	795	5*z*	222	95	221	245	223	330	670
6*x*	323	180	123	275	223	290	745	9*z*	332	215	333	300	331	255	770
3*y*	321	155	331	180	311	160	495	3*x*	213	185	313	145	113	150	480
9*y*	323	120	313	70	333	100	290	9*x*	333	50	133	45	233	105	200
7*y*	113	100	123	170	133	65	335	8*z*	322	155	323	125	321	30	310
9*x*	233	55	333	145	133	40	240	7*x*	131	65	331	130	231	55	250
1*x*	111	35	311	45	211	55	135	2*z*	122	85	123	55	121	110	250
7*y*	123	140	133	45	113	15	200	9*z*	331	130	332	40	333	45	215
1*x*	111	85	211	65	311	55	205	3*z*	131	45	132	60	133	15	120
1*z*	112	80	111	115	113	165	360	3*x*	313	0	213	70	113	65	135
1*y*	121	180	111	255	131	290	725	8*y*	223	285	213	270	233	185	740
5*x*	222	150	122	55	322	50	255	1*y*	111	210	131	265	121	185	660
8*x*	332	130	132	215	232	155	500	2*y*	211	95	221	95	231	155	345
4*x*	121	210	221	90	321	95	395	4*z*	213	160	212	140	211	125	425
7*z*	311	140	312	195	313	310	645	1*z*	111	210	112	290	113	325	825
4*y*	132	230	122	220	112	310	760	4*z*	211	230	213	155	212	195	580
3*z*	132	245	133	315	131	215	775	8*x*	132	160	232	285	332	230	675
6*z*	232	185	233	220	231	175	580	7*z*	311	275	313	185	312	130	590
2*z*	121	190	122	160	123	110	460	8*z*	323	155	321	150	322	240	545

TABLE 9.—YIELDS OF VARIETIES COLLECTED BY GROUPS AND FOR ALL GROUPS AND CALCULATION OF CORRECTED VARIETY MEANS. THREE DIMENSIONAL PSEUDO-FACTORIAL EXPERIMENT WITH THREE GROUPS OF SETS

w =		1				2				3				$X_{uv\cdot}$			
	u	1	2	3	$X_{\cdot vw}$	1	2	3	$X_{\cdot vw}$	1	2	3	$X_{\cdot vw}$	1	2	3	$X_{\cdot v\cdot}$
	v																
	1	120	120	100	340	585	675	600	1860	215	255	145	615	920	1050	845	2815
X	2	465	320	330	1115	295	500	270	1065	565	620	355	1540	1325	1440	955	3720
	3	475	385	365	1225	375	440	360	1175	85	160	195	440	935	985	920	2840
$X_{u\cdot w}$		1060	825	795	2680	1255	1615	1230	4100	865	1035	695	2595	3180	3475	2720	9375
					$Y_{\cdot vw}$				$Y_{\cdot vw}$				$Y_{\cdot vw}$		$Y_{uv\cdot}$		$Y_{\cdot v\cdot}$
	1	465	420	415	1300	620	485	440	1545	115	510	165	790	1200	1415	1020	3635
Y	2	365	410	325	1100	415	510	620	1545	310	570	260	1140	1090	1490	1205	3785
	3	555	455	450	1460	545	630	465	1640	110	540	390	1040	1210	1625	1305	4140
$Y_{u\cdot w}$		1385	1285	1190	3860	1580	1625	1525	4730	535	1620	815	2970	3500	4530	3530	11560
					$Z_{\cdot vw}$				$Z_{\cdot vw}$				$Z_{\cdot vw}$		$Z_{uv\cdot}$		$Z_{\cdot v\cdot}$
	1	325	355	415	1095	370	335	325	1030	490	315	495	1300	1185	1005	1235	3425
Z	2	300	380	180	860	245	325	395	965	165	600	280	2345	710	1305	855	2870
	3	260	505	385	1150	305	455	255	1015	330	435	345	1110	895	1395	985	3275
$Z_{u\cdot w}$		885	1240	980	3105	920	1115	975	3010	985	1350	1120	3455	2790	3705	3075	9570
			T_{uvw}		$T_{\cdot vw}$		T_{uvw}		$T_{\cdot vw}$		T_{uvw}		$T_{\cdot vw}$		$T_{uv\cdot}$		$T_{\cdot v\cdot}$
X	1	910	895	930	2735	1575	1495	1365	4435	820	1080	805	2705	3305	3470	3100	9875
Y	2	1130	1110	835	3075	955	1335	1285	3575	1040	1790	895	3725	3125	4235	3015	10375
Z	3	1290	1345	1200	3835	1225	1525	1080	3830	525	1135	930	2590	3040	4005	3210	10255
$T_{u\cdot w}$		3330	3350	2965	9645	3755	4355	3730	11840	2385	4005	2630	9020	9470	11710	9325	30505
			$T_{uvw}/3n$		$C_{\cdot vw}$		$T_{uvw}/3n$		$C_{\cdot vw}$		$T_{uvw}/3n$		$C_{\cdot vw}$		$C_{uv\cdot}$		
	1	151.667	149.167	155.000	+57.176	262.500	249.167	227.500	−22.268	136.667	180.000	134.167	+33.426	− 6.296	+ 0.741	−27.592	
	2	188.333	185.000	139.167	+ 1.574	159.167	222.500	214.167	+19.630	173.333	298.333	149.167	−15.787	+28.287	− 3.009	+ 1.713	
	3	215.000	224.167	200.000	+24.491	204.167	254.167	180.000	+28.518	87.500	189.167	155.000	+55.324	+10.509	−16.898	− 3.704	
$C_{u\cdot w}$		−25.972	−17.083	−19.861		−53.380	−40.463	−49.491		+34.120	−11.296	+17.592					
			t_{uvw}		$t_{\cdot vw}$		t_{uvw}		$t_{\cdot vw}$		t_{uvw}		$t_{\cdot vw}$		$t_{uv\cdot}$		
	1	176.575	190.001	164.723	177.100	180.556	187.177	128.149	165.294	197.917	202.871	157.593	186.127	185.016	193.350	150.155	
	2	192.222	166.482	122.593	160.432	153.704	198.658	186.019	179.460	219.953	268.241	152.685	213.626	188.626	211.127	153.766	
	3	224.028	214.677	200.926	213.210	189.814	225.324	155.323	190.154	187.453	216.297	224.212	209.321	200.432	218.766	193.487	
$t_{u\cdot w}$		197.608	190.387	162.747		174.691	203.720	156.497		201.774	229.136	178.163					

mple III.—Three Dimensional Pseudo-Factorial Experiment with Three Groups of Sets

Varieties in each set (p) = 3.

Varieties (v) = p^3 = 27, designated by numbers uvw as follows:

Group X(.vw)				*Group Y(u.w)*				*Group Z(uv.)*			
Set No.				Set No.				Set No.			
1	111	211	311	1	111	121	131	1	111	112	113
2	112	212	312	2	211	221	231	2	121	122	123
3	113	213	313	3	311	321	331	3	131	132	133
4	121	221	321	4	112	122	132	4	211	212	213
5	122	222	322	5	212	222	232	5	221	222	223
6	123	223	323	6	312	322	332	6	231	232	233
7	131	231	331	7	113	123	133	7	311	312	313
8	132	232	332	8	213	223	233	8	321	322	323
9	133	233	333	9	313	323	333	9	331	332	333

In the sets of *Group X* vw are constant, in *Group Y* uw are constant, and in *Group Z* uv are constant.

Sets (s) = $3p^2$ = 27
Replications of each group (n) = 2
Complete replications (r) = $3n$ = 6
Total number of blocks (b) = $3np^2$ = 54.
Total number of plots (N) = $3np^3$ = 162.

After the distribution of the blocks over the field and the randomization of the varieties within the blocks, we have such an arrangement as is shown in Table 8 in which the individual plot yields corresponding to the varieties are given. In this case the blocks were distributed over the whole field but it would be more convenient to keep them together in complete replications.

The variety yields are then collected by groups and for all groups as in Table 9. This table contains also the calculations of the corrected variety means as described on page 19. It is important to study this table carefully in order to be able to locate the correct totals for the calculation of the correction terms. Thus:

$$C_{\cdot vw} = \frac{1}{6np^2}(pT_{\cdot vw} - 3pX_{\cdot vw} - T_{\cdot v \cdot} + 3Y_{\cdot v \cdot})$$

$$\therefore \quad C_{\cdot 11} = \frac{1}{108}(3 \times 2735 - 9 \times 340 - 9875 + 3 \times 3635) = 57.176$$

$$\text{And} \quad C_{1\cdot 1} = \frac{1}{108}(3 \times 3330 - 9 \times 1385 - 9645 + 3 \times 3105) = -25.972$$

$$C_{11\cdot} = \frac{1}{108}(3 \times 3305 - 9 \times 1185 - 9470 + 3 \times 3180) = -6.296$$

Having obtained all of the correction terms we check by obtaining the total. In this case the total comes to + .001 which is a sufficiently close check.

The corrected means are then calculated by adding the corresponding correction terms to the actual means. Thus:

$$t_{111} = 151.667 + 57.176 - 25.972 - 6.296 = 176.575.$$

To obtain the sum of squares for varieties we average the corrected means in three directions to give $t_{\cdot vw}$, $t_{u\cdot w}$ and $t_{uv\cdot}$. Thus:

$$t_{\cdot 11} = \frac{1}{3}(176.575 + 190.001 + 164.723) = 177.100$$

$$t_{1\cdot 1} = \frac{1}{3}(176.575 + 192.222 + 224.028) = 197.608$$

$$t_{11\cdot} = \frac{1}{3}(176.575 + 180.556 + 197.917) = 185.016.$$

The sum of squares for varieties is then given by

$$\Sigma(t_{uvw} \cdot T_{uvw}) - [\,\Sigma(X_{\cdot vw} \cdot t_{\cdot vw}) + \Sigma(Y_{u\cdot w} \cdot t_{u\cdot w}) + \Sigma(Z_{uv\cdot} \cdot t_{uv\cdot})],$$

which in this case is

Varieties (SS) = 5,847,432.06 − 5,754,971.44 = 92,460.62.

After calculating the total and the block sum of squares from Table 8 we can set up the analysis of variance.

Analysis of Variance

Three dimensional pseudo-factorial experiment with three groups of sets

	SS	*DF*	*MS*	*F*	*5% pt.*
Blocks	1,154,025	53			
Varieties	92,461	26	3556	1.23	1.62
Error	236,872	82	2889		
Total	1,483,358	161			

The variances and standard errors for comparing the varieties are as follows. It will be noted that such comparisons now fall into three groups that can be determined from the variety numbers.

$$V(t_{211} - t_{111}) = \frac{2s^2}{3np^2}(p^2+p+1) = \frac{2 \times 2889}{54} \times 13 = 1391 \qquad SE = \sqrt{1391} = 37.30$$

$$V(t_{122} - t_{111}) = \frac{s^2}{3np^2}(2p^2+3p+4) = \frac{2889}{54} \times 31 = 1658 \qquad SE = \sqrt{1658} = 40.72$$

$$V(t_{222} - t_{111}) = \frac{s^2}{3np^2}(2p^2+3p+6) = \frac{2889}{54} \times 33 = 1766 \qquad SE = \sqrt{1766} = 42.02$$

And the mean variance of all comparisons is

$$V_m = \frac{s^2}{3n}\left(\frac{2p^2+5p+11}{p^2+p+1}\right) = \left(\frac{2889}{6} \times \frac{44}{13}\right) = 1630 \qquad SE = \sqrt{1630} = 40.37.$$

INCOMPLETE RANDOMIZED BLOCK EXPERIMENTS

The types of experiments previously discussed result in the varieties being apportioned into sets in such a way that the comparisons between pairs of varieties cannot all be made with equal precision. The difference between the precision of these comparisons is not great and therefore the methods cannot be criticized severely in this regard. However by an extension of the principle introduced in the discussion of Two Dimensional Experiments with Three Groups of Sets, Yates (8) has devised a method in which all comparisons are of equal precision, and there is the added advantage that the procedure of analysis is very much simplified. Although this method according to previous terminology would be referred to as the Two Dimensional Pseudo-Factorial method with all possible Groups of Sets we shall follow Yates and refer to it as the Incomplete Randomized Block method.

If we have 9 varieties represented by the following set of numbers:

1111	1232	1323
2122	2213	2331
3133	3221	3312 ,

we can arrange them in four groups of 3 sets each as follows where the first number represents the set in the first group, the second number the set of the second group, and so forth for the four groups:

Group 1			*Group 2*			*Group 3*			*Group 4*		
1111	1232	1323	1111	2122	3133	1111	2213	3312	1111	2331	3221
2122	2213	2331	1232	2213	3221	1323	2122	3221	1232	2122	3312
3133	3221	3312	1323	2331	3312	1232	2331	3133	1323	2213	3133

In these 12 sets any one variety occurs once and once only with every other variety. We cannot make up any more sets, therefore, unless a particular pair of varieties occurs more than once in the same set. Also if we have less than 12 sets it is obvious that certain pairs of varieties will not occur in the same set. Having reached the limit for the number of groups it follows therefore without algebraic proof that all comparisons will be of equal precision. We can now illustrate the practical possibilities of such an arrangement and the method of analysis.

In the first place only certain numbers of varieties can be arranged in sets such that each variety occurs once in the same set with every other variety. If p is the number of varieties in a set and v the total number of varieties, then if $v = p^2$ or $(p^2 - p + 1)$ we can demonstrate that for certain values of p the varieties can be distributed into sets as described above.

Referring to the arrangement of 9 varieties into 12 sets with 3 in each set, we note that this arrangement represents the case where $v = p^2$. Also the last two figures in the square of 9 numbers form a completely orthogonalized (3×3) square. Such squares are ordinarily known to mathematicians as Graeco-Latin Squares, and for a discussion of their properties the reader is referred to Fisher (1). The Graeco-Latin Square corresponding to the above would be

A_1	C_2	B_3
B_2	A_3	C_1
C_3	B_1	A_2

where we replace the first figure by Latin Letters and the second figure by subscripts. Fisher (1) has illustrated that for those Graeco-Latin Squares that are possible there are for a square of p^2 dimensions, $(p - 1)$ elements such as Latin Letters, subscripts, Greek Letters, etc.

Now for the type of Incomplete Randomized Block experiment where $v = p^2$ the first two groups of sets are made up from the rows and columns respectively of the variety numbers arranged in the form of a square and the remaining $(p - 1)$ groups of sets successively from those varieties corresponding to the same Latin Letter, the same subscript, the same Greek Letter, etc., of the super-imposed Graeco-Latin Square. Thus for p^2 varieties we must use $(p + 1)$ replications in order to make up an Incomplete Randomized Block experiment.

If the number of varieties is $p^2 - p + 1$ which is obviously equal to $(p - 1)^2 + p$ we can form the sets by first taking p of the numbers to make up one set and writing down the remaining $(p - 1)^2$ numbers in the form of a square. This square will generate p groups of sets which are compiled by allotting one of the first sets of p varieties to each of the p groups of $(p - 1)$ sets in turn.

This gives then a total of $(p^2 - p + 1)$ sets of p varieties. For example if we have 13 varieties which we shall represent by numbers as follows

01	02	03	04
	11	12	13
	21	22	23
	31	32	33

it is possible to arrange these in 13 sets of 4 each so that we have any one variety occurring once and once only with any other variety. To do this we first make up one set consisting of the varieties (01, 02, 03, 04). Then we arrange the other varieties in 12 sets of 3 and to each group we attach one of the varieties in the first set. The 13 sets as finally made up are as follows:

Set No.					Set No.				
1	01	02	03	04					
2	01	11	12	13	8	03	11	22	33
3	01	21	22	23	9	03	21	32	13
4	01	31	32	33	10	03	31	12	23
5	02	11	21	31	11	04	11	32	23
6	02	12	22	32	12	04	21	12	33
7	02	13	23	33	13	04	31	22	13

In general terms if p is the number of varieties in one set, the number of varieties $(v) = p^2 - p + 1$, the number of sets $(b) = p^2 - p + 1$, and the number of replications $(r) = p$.

There are other types that can be made up; for example, we can put $v = p^3$, and for 27 varieties we can use 13 replications and 117 blocks, but owing to the number of replications required this type is of lesser practical importance than the first two types discussed.

On the basis of the following relations between v, r, p and b

Let $v = p^2$	Then $r = p + 1$	$b = p(p + 1)$
" $v = p^2 - p + 1$	" $r = p$	$b = v$
" $v = p^3$	" $r = p^2 + p + 1$	$b = p^2(p^2 + p + 1)$,

we can make up a table showing some of the possible arrangements by the Incomplete Randomized Block method.

TABLE 10.—SOME OF THE POSSIBLE ARRANGEMENTS WITH DIFFERENT NUMBERS OF VARIETIES USING THE INCOMPLETE RANDOMIZED BLOCK METHOD

	Type $v = p^2$			Type $v = p^2-p+1$			Type $v = p^3$		
p	v	b	r	v	b	r	v	b	r
3†	9	12	4	7	7	3	27	117	13
4	16	20	5	13	13	4	64	336	21
5†	25	30	6	21	21	5			
6	$\boxed{36^*}$	42	7	31	31	6			
7†	49	56	8	$\boxed{43}$	43	7			
8	64	72	9	57	57	8			
9	81	90	10	73	73	9			
10	100	110	11	91	91	10			
11†	121	132	12	111	111	11			
12	144	156	13	133	133	12			
Orthogonalized square required		p^2			$(p-1)^2$				

*Variety numbers in square impossible because completely orthogonalized (6×6) square does not exist.
†Sets can be written down by rule.

Making up the Sets

The first problem in the construction of an Incomplete Randomized Block experiment is to write out the sets. As pointed out above the problem is simple if we have the corresponding completely orthogonalized square. These are not necessary, however, for the cases where p is a prime number as the sets can be written down by rule. This method will be illustrated first.

Suppose $p = 5$ and the experiment is of the type $v = p^2$. We write down the variety numbers in a (5×5) square using any convenient notation. We shall use here the notation already introduced where each variety is represented by the number uv where u represents the row and v the column of the square. Our square is then as follows:

11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45
51	52	53	54	55

The first two groups of sets are written down from the rows and columns. The rule for writing the other four groups of sets is to start with Group 3 and write in the rows the numbers that occur in the diagonals of the original square. Group 4 then results from writing in the rows the numbers in the diagonals of the square for Group 3. This procedure is continued until we reach Group 6. If we apply this procedure to Group 6 the original square is regenerated and this may be used as a check on the work. The six groups as finally written out are as in Table 11.

TABLE 11.—THE SIX GROUPS OF SETS FOR AN INCOMPLETE RANDOMIZED BLOCK EXPERIMENT WHERE $p = 5$

Group 1

11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45
51	52	53	54	55

Group 2

11	21	31	41	51
12	22	32	42	52
13	23	33	43	53
14	24	34	44	54
15	25	35	45	55

Group 3

11	22	33	44	55
21	32	43	54	15
31	42	53	14	25
41	52	13	24	35
51	12	23	34	45

Group 4

11	32	53	24	45
21	42	13	34	55
31	52	23	44	15
41	12	33	54	25
51	22	43	14	35

Group 5

11	42	23	54	35
21	52	33	14	45
31	12	43	24	55
41	22	53	34	15
51	32	13	44	25

Group 6

11	52	43	34	25
21	12	53	44	35
31	22	13	54	45
41	32	23	14	55
51	42	33	24	15

If the experiment is of the type $v = p^2 - p + 1$ and $(p - 1)$ is a prime number the same method may be used. For example if we have 31 varieties, $p = 6$, and $(p - 1) = 5$. Using the same notation as above with the addition of 6 varieties represented by 01, 02, 03, 04, 05, 06, we put these 6 together in one set and proceed to make up the others attaching 01, to all of the sets of Group 1, 02 to all of the sets of Group 2, and so forth.

For experiments of the $v = p^2$ type, if p is not a prime number the orthogonal square must be used, and also for experiments of the $v = p^2 - p + 1$ type where $(p - 1)$ is not a prime number. The (6×6) orthogonal square is impossible and those greater than (9×9) have not yet been worked out. The squares for $p = 4$, and 9 are reproduced here. The (9×9) square was very kindly supplied by Dr. R. A. Fisher. A more complete set is being given by Dr. Fisher in the second edition of *Design of Experiments*.

TABLE 12.—ORTHOGONAL SQUARES

	111	234	342	423
4×4	222	143	431	314
	333	412	124	241
	444	321	213	132

	1111	2347	3274	4732	5968	6895	7423	8659	9586
	1111	9658	5896	8965	4273	3427	6589	2734	7342
	2222	3158	1385	5813	6749	4976	8531	9467	7694
	2222	7469	6974	9746	5381	1538	4697	3815	8153
	3333	1269	2196	6921	4857	5784	9612	7548	8475
	3333	8547	4785	7854	6192	2619	5478	1926	9261
	4444	5671	6517	7165	8392	9238	1756	2983	3829
	4444	3982	8239	2398	7516	6751	9823	5167	1675
9×9	5555	6482	4628	8246	9173	7319	2864	3791	1937
	5555	1793	9317	3179	8624	4862	7931	6248	2486
	6666	4593	5439	9354	7281	8127	3945	1872	2718
	6666	2871	7128	1287	9435	5943	8712	4359	3594
	7777	8914	9841	1498	2635	3562	4189	5326	6253
	7777	6325	2563	5632	1849	9184	3256	8491	4918
	8888	9725	7952	2579	3416	1643	5297	6134	4361
	8888	4136	3641	6413	2957	7295	1364	9572	5729
	9999	7836	8763	3687	1524	2451	6378	4215	5142
	9999	5214	1452	4521	3768	8376	2145	7683	6837

The use of these orthogonal squares for writing down the sets of varieties will be illustrated for the case $(p - 1) = 4$, $v = 21$. We first write down the numbers for the varieties as follows:

01	02	03	04	05
	11	12	13	14
	21	22	23	24
	31	32	33	34
	41	42	43	44

The first set is (01, 02, 03, 04, 05) and the next 8 sets can be written down from the rows and columns of the (4×4) square.

01	11	12	13	14
01	21	22	23	24
01	31	32	33	34
01	41	42	43	44

Group 1

02	11	21	31	41
02	12	22	32	42
02	13	23	33	43
02	14	24	34	44

Group 2

For the other three groups of 4 sets each we make use of the orthogonal square given in Table 12. Assuming the 16 variety numbers arranged in a square and superimposed on the orthogonal square we note, considering the first of the three digit numbers only, that 1 corresponds with the variety numbers 11, 22, 33, 44; 2 with 21, 12, 43, 34; 3 with 31, 42, 13, 24; and 4 with 41, 32, 23, 14. These are the sets of the third group and we make up two more groups by using the second and third figures of the orthogonal square. Groups 3, 4, and 5 are finally as follows:

03	11	22	33	44
03	21	12	43	34
03	31	42	13	24
03	41	32	23	14

Group 3

04	11	32	43	24
04	21	42	33	14
04	31	12	23	44
04	41	22	13	34

Group 4

05	11	42	23	34
05	21	32	13	44
05	31	22	43	14
05	41	12	33	24

Group 5

Laying out the Field

After writing out the sets these are distributed over the field and the varieties randomized within each set. If the experiment is of the $v = p^2$ type the groups of sets may be kept together as complete replications, but if it is of the $v = (p^2 - p + 1)$ type the groups do not correspond to complete replications and there is in fact no way in which the sets can be arranged together in groups to form complete replications.

Analysis of the Data

The data are best collected as in Table 14 of Example IV. From this table we obtain the block totals directly and recopy the results as in Table 15 to obtain the variety totals. The next step is to obtain the quantities Σ_{uv} which are totals for all of the blocks containing the variety *uv*. The performance of a variety is to be measured by its yield in relation to the yields of all of the other varieties. For example, for an experiment with 9 varieties the yields of individual plots may be set out as in Table 13 by varieties and Incomplete blocks. The block totals are represented by B and the variety totals by T_{uv}. It would obviously be unfair to compare varieties 11 and 12 by means of their actual totals as they only occur together in Block 1. The other plots are all in different

TABLE 13.—TWO-WAY TABLE BY VARIETIES AND BLOCKS FOR THE YIELDS OF SINGLE PLOTS IN AN INCOMPLETE RANDOMIZED BLOCK EXPERIMENT WITH 9 VARIETIES

	11	12	13	21	22	23	31	32	33		
1	x	x	x							B_1	
2				x	x	x				B_2	T_{uv} = Variety totals
3							x	x	x	B_3	
4	x			x			x			B_4	$B_1 ... B_{12}$ = Block totals
5		x			x			x		B_5	
6			x			x			x	B_6	Then
7	x				x				x	B_7	$\Sigma_{11} = (B_1 + B_4 + B_7 + B_{11})$
8			x	x				x		B_8	
9		x				x	x			B_9	
10		x		x					x	B_{10}	
11	x					x		x		B_{11}	
12			x		x		x			B_{12}	
	T_{11}	T_{12}	T_{13}	T_{21}	T_{22}	T_{23}	T_{31}	T_{32}	T_{33}	$T_{..}$	

blocks and consequently the variety totals are partially confounded with block effects. It is perfectly fair, however, to take as a measure of the performance of Variety 11 the difference between its weighted total and the total of all of the other plots in the same blocks. The other 8 plots are made up of one plot each of the other 8 varieties. If two such measures say for Varieties 11 and 12 are compared, the difference between them is due entirely to the two varieties as the block effect is completely eliminated. If we represent the variety total by T_{uv} and the yields of all of the remaining plots by A, the difference required is $a_{uv} = [(p - 1)T_{uv} - A]$ but this is obviously the same as $(pT_{uv} - \Sigma_{uv})$ where Σ_{uv} is the total of all the blocks containing the variety *uv*. In the example above $a_{11} = [4T_{11} - (B_1 + B_4 + B_7 + B_{11})]$ or $a_{11} = (4T_{11} - \Sigma_{11})$.

The sum of squares for varieties is now given by the simple formula

$$\text{Varieties } (SS) = \frac{\Sigma(pT_{uv} - \Sigma_{uv})^2}{vp}.$$

Or if we let $$t_{uv} = \frac{pT_{uv} - \Sigma_{uv}}{v} \text{ we have}$$

$$\text{Varieties } (SS) = \frac{v}{p}\Sigma(t_{uv}^2),$$

where t_{uv} is the actual quantity that might be used to compare varieties, or if it is more convenient $(t_{uv} + m)$where m is the general mean of the experiment.

In order to obtain the values Σ_{uv} it is not convenient in a large experiment to make up a two-way table as above, so it is suggested that the sets be written out as in Table 16 and the block totals written down opposite each set. Then to obtain Σ_{42} for example it is only necessary to find 42 in each group and add the corresponding block totals. This is comparatively easy as 42 occurs in the same column in all of the groups except Group 2. For Group 2, however, the number of the set is given by adding 5 to the last figure of the variety number. Thus 42 occurs in set 4 (first number), set 7 (2 + 5), and sets 13, 17, 21, and 30.

The sum of squares for blocks is obtained directly and the error sum of squares by differences. The form of the analysis for an experiment of each of the two types is given below.

Type $v = p^2$	
Blocks	$p(p + 1) - 1$
Varieties	$p^2 - 1$
Error	$(p - 1)(p^2 - 1)$
Total	$p^2(p + 1) - 1$

Type $v = p^2 - p + 1$	
Blocks	$p(p - 1)$
Varieties	$p(p - 1)$
Error	$(p - 1)^3$
Total	$(p - 1)(p^2 + 1)$

Finally in comparing varieties by their values of $t_{uv} + m$ (corrected variety means) the variance of a difference is

$$\frac{2s^2}{r}\left(\frac{p + 1}{p}\right) \text{ for the type } v = p^2$$

$$\frac{2s^2}{r}\left(\frac{p^2}{p^2 - p + 1}\right) \text{ for the type } v = p^2 - p + 1.$$

Example IV.—Incomplete Randomized Block Experiment

Varieties in each set $(p) = 6$
Varieties $(v) = (p^2 - p + 1) = 31$
Incomplete blocks $(b) = (p^2 - p + 1) = 31$
Replications $(r) = p = 6$
Total number of plots $(N) = p(p^2 - p + 1) = 186.$

Complete instructions for numbering the varieties and writing out the sets are given in the text. The blocks may be distributed over the field as shown in Table 14, or they may be retained in the same order as they are made up. The yields corresponding to individual variety plots and the block totals are given in this table.

The second step in the calculations is to prepare Table 15, in order to collect the yields by varieties and calculate the variety totals T_{uv}. Corresponding to each variety total we have also the quantities Σ_{uv}. These are most easily calculated as described in the text by the preparation of Table 16 giving block totals arranged in order of the block numbers. For each variety it is then a simple matter to sum the block totals giving the appropriate value of Σ_{uv}.

The variety sum of squares is obtained by summing the squares of the quantities $(pT_{uv} - \Sigma_{uv})$ and dividing this sum by $(vp) = 186$ in this example.

Varieties $(SS) = \Sigma(pT_{uv} - \Sigma_{uv})^2/vp = 1{,}617{,}224$. Finally the analysis of variance is

Analysis of Variance

Incomplete block experiment for 31 varieties and 6 replications

	SS	*DF*	*MS*	*F*	5% *pt.*
Blocks	1,083,491	30	36,116	10.5	1.53
Varieties	103,977	30	3,466	1.01	1.53
Error	429,756	125	3,438		
Total	1,617,224	185			

Table 15 also shows the corrected variety means. These are calculated by dividing the quantities $(pT_{uv} - \Sigma_{uv})$ by v, the number of varieties, and adding the general mean.

The variance and standard error for comparing any two varieties by their corrected means is

$$V_m = \frac{2 \times 3438}{6} \times \frac{36}{31} = 1330.8 \quad SE = \sqrt{1330.8} = 36.48.$$

Table 14.—Location of the varieties in the field, corresponding plot yields, and block totals. Symmetrical incomplete block experiment with 31 varieties and 6 Replications

Set No.	Var.	Yield	Var.	Yield	Var.	Yield	Var.	Yield	Var.	Yield	Var.	Yield	Block Totals
1	11	315	13	370	01	360	14	265	12	355	15	345	2010
2	23	245	22	185	21	160	01	285	24	355	25	240	1470
3	01	325	33	315	32	300	35	240	31	220	34	350	1750
4	45	360	43	230	42	225	01	270	41	255	44	170	1510
5	01	175	53	290	51	330	54	220	52	220	55	265	1500
6	31	195	11	310	21	315	02	215	41	330	51	270	1635
7	22	290	52	95	02	140	32	330	12	410	42	235	1500
8	13	255	23	375	43	305	33	255	02	235	53	230	1655
9	54	275	44	245	34	140	24	270	14	230	02	135	1295
10	45	95	35	245	02	330	25	235	15	200	55	285	1390
11	44	180	11	275	33	290	55	155	03	180	22	160	1240
12	03	120	32	70	21	100	15	100	43	170	54	65	625
13	53	55	42	145	31	40	25	35	03	45	14	55	375
14	24	140	13	45	35	15	03	85	52	65	41	55	405
15	45	80	23	115	34	165	03	85	51	55	12	120	620
16	32	215	11	300	45	255	24	185	04	145	53	150	1250
17	13	50	34	45	55	105	42	155	21	125	04	30	510
18	23	65	15	130	44	55	31	85	04	55	52	110	500
19	25	130	33	40	41	45	12	45	54	60	04	15	335
20	35	−5	04	70	22	65	43	35	14	255	51	80	500
21	05	180	11	255	23	290	42	285	35	270	54	185	1465
22	21	150	52	55	14	50	45	210	33	265	05	185	915
23	55	130	24	215	12	155	31	95	05	95	43	155	845
24	15	210	41	90	53	95	22	160	05	140	34	125	820
25	32	140	05	195	13	310	51	195	25	130	44	285	1255
26	11	210	34	290	43	325	25	230	52	220	06	310	1585
27	12	230	44	155	35	195	53	245	06	315	21	215	1355
28	13	160	31	285	54	230	22	185	45	220	06	175	1255
29	14	275	55	185	06	130	32	190	41	160	23	110	1050
30	15	155	42	150	24	240	06	130	33	145	51	125	945
31	01	220	05	215	06	195	03	240	02	295	04	230	1395
													34960

TABLE 15.—YIELDS OF SINGLE PLOTS BY VARIETIES, VARIETY TOTALS, VALUES OF Σ_{uv} AND THE CORRECTED MEANS t_{uv}. SYMMETRICAL INCOMPLETE BLOCK EXPERIMENT WITH 31 VARIETIES AND 6 REPLICATIONS

Var. No.	Single Plot Yields						T_{uv}	Σ_{uv}	$pT_{uv}-\Sigma_{uv}$	t_{uv}
01	360	285	325	270	175	220	1635	9635	175	193.6
02	215	140	235	135	330	295	1350	8870	− 770	163.2
03	180	120	45	85	85	240	755	4660	− 130	183.8
04	145	30	55	15	70	230	545	4490	−1220	148.6
05	180	185	95	140	195	215	1010	6695	− 635	167.5
06	310	315	175	130	130	195	1255	7585	− 55	186.2
11	315	310	275	300	255	210	1665	9185	805	214.0
12	355	410	120	45	155	230	1315	6665	1225	227.5
13	370	255	45	50	310	160	1190	7090	50	189.6
14	265	230	55	255	50	275	1130	6145	635	208.5
15	345	200	100	130	210	155	1140	6290	550	205.7
21	160	315	100	125	150	215	1065	6510	− 120	184.1
22	185	290	160	65	160	185	1045	6785	− 515	171.4
23	245	375	115	65	290	110	1200	6760	440	202·2
24	355	270	140	185	215	240	1405	6210	2220	259.6
25	240	235	35	130	130	230	1000	6410	− 410	174.8
31	220	195	40	85	95	285	920	6360	− 840	160.9
32	300	330	70	215	140	190	1245	7430	40	189.3
33	315	255	290	40	265	145	1310	6840	1020	220.9
34	350	140	165	45	125	290	1115	6580	110	191.9
35	240	245	15	−5	270	195	960	6865	−1105	152.4
41	255	330	55	45	90	160	935	5755	− 145	183.3
42	225	235	145	155	285	150	1195	6305	865	215.9
43	230	305	170	35	155	325	1220	6720	600	207.4
44	170	245	180	55	285	155	1090	7155	− 615	168.2
45	360	95	80	255	210	220	1220	6940	380	200.3
51	330	270	55	80	195	125	1055	6455	− 125	184.0
52	220	95	65	110	55	220	765	6405	−1815	129.4
53	290	230	55	150	95	245	1065	6955	− 565	169.8
54	220	275	65	60	185	230	1035	6475	− 265	179.4
55	265	285	155	105	130	185	1125	6535	215	194.9
	8215	7495	3680	3555	5490	6525	34960	209760	0	

$$m = \frac{34960}{186} = 188.0$$

TABLE 16.—SETS ARRANGED IN ORDER OF NUMBERS WITH CORRESPONDING BLOCK TOTALS. INCOMPLETE RANDOMIZED BLOCK EXPERIMENT

Set No.							Block Totals	Set No.							Block Totals
1	01	11	12	13	14	15	2010	16	04	11	32	53	24	45	1250
2	01	21	22	23	24	25	1470	17	04	21	42	13	34	55	510
3	01	31	32	33	34	35	1750	18	04	31	52	23	44	15	500
4	01	41	42	43	44	45	1510	19	04	41	12	33	54	25	335
5	01	51	52	53	54	55	1500	20	04	51	22	43	14	35	500
6	02	11	21	31	41	51	1635	21	05	11	42	23	54	35	1465
7	02	12	22	32	42	52	1500	22	05	21	52	33	14	45	915
8	02	13	23	33	43	53	1655	23	05	31	12	43	24	55	845
9	02	14	24	34	44	54	1295	24	05	41	22	53	34	15	820
10	02	15	25	35	45	55	1390	25	05	51	32	13	44	25	1255
11	03	11	22	33	44	55	1240	26	06	11	52	43	34	25	1585
12	03	21	32	43	54	15	625	27	06	21	12	53	44	35	1355
13	03	31	42	53	14	25	375	28	06	31	22	13	54	45	1255
14	03	41	52	13	24	35	405	29	06	41	32	23	14	55	1050
15	03	51	12	23	34	45	620	30	06	51	42	33	24	15	945
								31	06	01	02	03	04	05	1395
															34960

DISCUSSION AND COMPARISON OF VARIOUS METHODS

In the selection of an experimental method for field plot work the factor of primary importance is efficiency. If a given method promises to bring about an increase in efficiency it must be very seriously considered and any increase in the cost of operation carefully balanced against the improvement in the results. The Pseudo-Factorial and Incomplete Randomized Block methods require in certain cases more replication and in all cases a slightly greater expenditure of labour in analysing the results. With some of these methods the increased amount of computation is practically negligible. In others it may amount to two or three extra days work for a computer as compared to the same test carried out in ordinary Randomized Blocks. This would apply only to large tests and in such cases the computational work would be small as compared to the total amount of labour taken up in conducting the test in the field, and following up with additional laboratory tests. This point would seem to warrant considerable emphasis. If a given sum is to be expended in conducting a variety test we can regard the test as giving a certain number of units of information and ascertain the actual cost per unit. Suppose we say that a test with 100 varieties will give us 100 units of information and if the total cost is $1,000 the cost per unit is $10. Now if we use a more efficient method the expenditure will be increased slightly and in the case of substituting a Pseudo-Factorial experiment for Randomized Blocks the total cost may be increased to $1,050 but with careful planning it is quite possible that the efficiency will be increased by 50%. In other words we now obtain 150 units of information and the cost per unit is $1,050/150 = $7. This is particularly enlightening in view of the tendency on the part of some experimenters to regard statistical work as a kind of expensive luxury with which they can very well do without.

We can illustrate the gain in efficiency by the use of the new methods as compared to other forms of experimentation by a study of uniformity data. There is insufficient space here to present results for a number of cases but one example would seem to be worth while.

The data given by Wiebe (5) for yields of 15-foot rows of wheat were used. These were combined first in triple rows so as to give plots conforming very closely to those used in actual rod row trials. In this way two sets of yields were made up, each set occupying a rectangle 90 ft. × 108 ft., and consisting of 6 series of 36 plots. Assuming that we have 36 varieties to test we can replicate 6 times and make up an imaginary experiment in at least three different ways.

(1) A Latin Square of Groups. The varieties are divided into 6 groups of 6 each and the groups arranged in a (6 × 6) Latin Square.

(2) A (6 × 6) Pseudo-Factorial Experiment with Two Groups of Sets.

(3) An ordinary Randomized Block Experiment with 6 replications. Since we are concerned only with an estimate of the error we can disregard the varieties and pool all sums of squares that are not removed in error control. The analyses of variance by each of the three methods for the two sets of data are given in Table 17. The Randomized Blocks were made up by combining the plots in blocks 36 ft. × 45 ft.

To compare methods (1) and (3) we find the ratio of the mean squares and multiply by 7/9 which is the efficiency factor $(p + 1)/(p + 3)$ for Pseudo-Factorial experiments of this type. We get

$$\text{Set I} \quad \frac{42{,}748.7}{19{,}963.7} \times \frac{7}{9} = 166.5\% \qquad \text{Gain} = 66.5\%$$

$$\text{Set II} \quad \frac{27{,}916.7}{11{,}607.8} \times \frac{7}{9} = 187.0\% \qquad \text{Gain} = 87.0\%.$$

These data may show an exceptional gain in efficiency for the Pseudo-Factorial Method but they were not selected for this purpose. The comparison is perfectly fair in that the length-width proportions of the Randomized Blocks and the Incomplete Blocks are very nearly the same.

In considering method (2) it is of interest first to note the difference between the variances for comparing varieties within and between groups. These variances are

Table 17.—Analysis of variance by three methods of two sets of uniformity data

	Set I			Set II		
Randomized Blocks (1)						
	SS	*DF*	*MS*	*SS*	*DF*	*MS*
Blocks	8,089,477.4	5		2,751,650.8	5	
Error	8,977,221.6	210	42,748.7	5,862,507.8	210	27,916.7
Total	17,066,699.0	215		8,614,158.6	215	
Latin Square of Groups (2)						
	SS	*DF*	*MS*	*SS*	*DF*	*MS*
Rows	4,804,909.4	5		3,367,186.0	5	
Col's.	5,801,946.9	5		1,918,096.9	5	
Between Groups	2,866,380.2	25	114,655.2	1,239,466.2	25	49,578.6
Within Groups	3,593,462.5	180	19,963.7	2,089,409.5	180	11,607.8
Total	17,066,699.0	215		8,614,158.6	215	
Pseudo-Factorial Experiment (3)						
	SS	*DF*	*MS*	*SS*	*DF*	*MS*
Blocks	13,473,236.5	35		6,524,749.1	35	
Error	3,593,462.5	180	19,963.7	2,089,409.5	180	11,607.8
Total	17,066,699.0	215		8,614,158.6	215	

$$\text{Set I } V\text{ (between)} = \frac{2}{6}\left(\frac{1}{6}\ 114{,}655.2 + \frac{5}{6}\ 19{,}963.7\right) = 11915.2$$

$$V\text{ (within)} = \frac{2}{6}\,(19{,}963.7) = 6654.6$$

$$\text{Set II } V\text{ (between)} = \frac{2}{6}\left(\frac{1}{6}\ 49{,}578.6 + \frac{5}{6}\ 11{,}607.8\right) = 5978.8$$

$$V\text{ (within)} = \frac{2}{6}\,(11{,}607.8) = 3869.3$$

In Set I the ratio of the two variances is 179.0% so that the comparisons within groups are 79.0% more efficient than those between groups. In Set II the ratio is 154.5%. Now in the Pseudo-Factorial Experiment we also have two kinds of comparisons depending on whether or not the varieties compared occur in the same set, but the difference in the efficiency of the comparisons is fixed by the type of the experiment and is therefore independent of the data. The ratio of the two variances in this case is 8/7 = 114.3%. Remembering that p is fairly small and that the type of Pseudo-Factorial chosen has a greater ratio of the two variances than any of the other types it is obvious that Pseudo-Factorials in general are quite evenly balanced.

We can of course compare the average variance for single comparisons according to method (1) with a similar average for method (2). It we take any one of the 36 varieties, by method (1) there are 5 comparisons that can be made with varieties in the same group and 30 comparisons with varieties in different groups. Thus the average variance for all comparisons in the two Sets will be

$$\text{Set I} \quad (5 \times 6654.6 + 30 \times 11915.2)/35 = 11{,}163.7$$
$$\text{Set II} \quad (5 \times 3869.3 + 30 \times 5978.8)/35 = 5{,}677.4$$

The corresponding average variances for the Pseudo-Factorial method are

$$\text{Set I} \quad 6654.6 \times \frac{9}{7} = 8{,}555.9$$

$$\text{Set II} \quad 3869.3 \times \frac{9}{7} = 4{,}974.8.$$

The gain in efficiency of the Pseudo-Factorial method over the Latin Square of Groups is 30.5% for Set I, and 14.1% for Set II. This coupled with the improved balance between the comparisons is therefore very favourable to the Pseudo-Factorial method.

Granted that the Pseudo-Factorial and Incomplete Randomized Block methods are more efficient in general for testing large numbers of varieties than any other methods that have yet been devised, the next point of interest is to decide which of the various types are most suitable for particular cases. It will have been noted that the Incomplete Randomized Blocks are the most desirable from the standpoint of simplicity of calculation, and in addition all comparisons are of equal precision. The drawback with this method is that with variety numbers exceeding 50 the number of replications required is more than most agronomists are accustomed to using and perhaps more than many can afford. The ideal situation would be to use the Incomplete Randomized Blocks for variety numbers up to 100, and beyond that to use one of the Pseudo-Factorial types. However, for numbers in excess of 50, if the experimenter feels that the required number of replications are more than can be handled conveniently, the Pseudo-Factorials are perfectly satisfactory. These are only possible of course for variety numbers that are a perfect square but this is not a serious difficulty with fairly large numbers of varieties as it is always possible to add a few extra varieties or "dummies" in order to bring the number up to a perfect square. Also for those who wish definitely to use other numbers than those listed here, Yates (6) has developed methods for laying out and analysing Pseudo-

Factorials in which the dimensions are not equal. Thus instead of a (12 × 12) Pseudo-Factorial for 144 varieties we might use a (12 × 11) for 132 varieties. These modifications however require additional computations and will be avoided if possible.

Beyond 100 varieties and up to 200 the Pseudo-Factorial method with two groups of sets would seem to be best adapted. The value of p is large enough so that the comparisons between varieties occurring once in the same set and between those not occurring in the same set are reasonably well equalized, and for all practical purposes one may use the average variance of such comparisons for all cases.

With still larger numbers the value of p may be too large for the greatest efficiency and the Three Dimensional Pseudo-Factorial method is recommended. The computations are somewhat laborious but a test of say 216 varieties would seem to warrant at least two or three days of calculation. This would be only a small proportion of the total labour involved in the experiment.

SUMMARY

To summarize the various practical features of Pseudo-Factorial and Incomplete Randomized Block experiments we may enumerate as follows, bringing out any additional points that have not previously been mentioned.

1. Increased efficiency over methods now in use. In the examples worked out by Yates (6) the increases ranged from 26 to 57%. In the example given here for two sets of data the increases in efficiency over Randomized Blocks were 66 and 87%. With a reasonable amount of care in arranging the shape and size of the plots so that the Incomplete Blocks are nearly square it would seem that an increase of 50% might be expected on the average.

2. Adaptability of the methods to irregularly shaped fields and to fields cut up by wide roadways. The Incomplete Block is the unit and its position with respect to any other Block is irrelevant.

3. The first replicate of the experiment when using all of the Pseudo-Factorial Methods and Incomplete Randomized Blocks of one type can be laid down so as to conform very closely to the systematic arrangements preferred by some experimenters. With 36 varieties for example we can divide the varieties into 6 groups according to time of maturity and we can lay down the blocks in order of the time of maturity of the varieties they contain. It is only necessary to randomize within the blocks.

4. The randomization process in laying out an experiment with a large number of varieties is easier than for a similar experiment conducted with Randomized Blocks. The varieties are first assigned to the sets in any order whatsoever. As each block is made up from a given set the varieties are randomized within the block.

5. Certain numbers of varieties cannot be set up in a Pseudo-Factorial or Incomplete Randomized Block experiment. This can usually be adjusted without a great deal of trouble by adding other varieties or "dummies". When a given number must be used and the methods described here cannot be adapted there are other possibilities such as the methods suggested by Yates (6) with Unequal Groups of Sets. These require however some additional computational labour.

6. The actual variety means being confounded with block effects are not used for comparing varieties directly. Instead the corrected variety means are calculated from which the block effects have been removed. Thus the yields of varieties as reported in final form are variety scores rather than actual yields.

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